1. Introduction


HYENA is based on a mathematical iteration process as follows:

- Read and prepare model data for Power system, Water series, Load demand etc for all operational years and water years.
- Start Iteration
  - **Strategic part.** Simplified description of System. Redefine load in water areas with information from last Iteration as:
    - Demand
    - + Losses
    - – Transmission into Water Area
    - + Transmission from Water Area
    - + Selling activity on the Spot Market
    Calculation of water values for water areas taking into account local reserve power in water area.
  - **Tactical part.** Simulation of the Power System base on calculated water values for water areas, detailed description of the Production System, Transmission System and Load Demand in Substations.
  - Analyse results. If convergence then leave
- Repeat
- Print out Results
- End
The convergence of the iteration process is very efficient so usually three iterations give satisfactory result. The chart shows typical five iterations, first four using LP and the last using LP/NLP. NLP gives better result in this case.

The iteration process used in HYENA does not calculate exact optimum solution which would take exponential time and computer resources. Instead the process finds an approximate solution trading optimality for efficiency. Such solutions, often loosely called heuristic methods or heuristics, seek to obtain good near-optimal solutions at relatively low computational costs sacrificing guarantee of optimality. The mathematical method used in HYENA, that iteratively improves the solution towards optimality, is in the literature referred to as metaheuristic. Example of this kind of methodology is the emerging technology of Ant Colony Optimization and Genetic Algorithms. From Mathematical Programming Glossary: “This is a general framework for heuristics in solving hard problems. The idea of ‘meta’ is that of level.”

The biggest disadvantage of this approach is in sensitivity analysis but by tuning the HYENA model properly it has proven not to be a major problem. All kinds of sensitivity analysis can be performed with satisfactory and stable results.

As an advantage with HYENA one may argue that simulations are considered closer to reality than optimization with perfect foresight.
2. **Hyena model Options**

Following are the most important options available in HYENA and they can be mixed in any combination.

**Optimization technique**

- Linear Programming (LP)
- Mixed Integer Linear Programming (NLP)
- Nonlinear Programming
- Mixed Integer Nonlinear Programming

**Constraints**

- Without Transmission Lines
- Without Transmission/FlowGates Constraints
- With Transmission Constraints in Lines
- With Transfer constraints in FlowGates

**Losses in transmission lines**

- Nonlinear or Linear Losses

**Time Unit**

- Week or Day

If NLP is used say for 5 iterations then in the first four iterations LP is used and in the last iteration the model first calculates an approximate solution before starting the NLP calculation. LP is 5-15 times faster than NLP depending on size of the model. Using the model for say 1 operational year, 51 water years, weekly time units 3 iterations, Linear Programming, and 30-40 hydro units takes ca 1 minute.

The chart shows typical results from HYENA for a small reservoir in a spotmarket environment. The figure shows swarm of reservoir curves of 50 water years with day as a time unit and using NLP. The thick water value curves give indication on when to buy or sell on the spot market. Between the curves there is neither buying nor selling, the system is in a waiting state.
Figure 2

Figure 3 on the other hand shows a swarm of reservoir curves for a large reservoir in a non-spotmarket environment, 51 water years and 5 operational years with a time unit of 1 day and using NLP. The reservoir is assumed starting operation in 01.09.2006.

Figure 3

Detailed information as in figure 3 gives important guidelines on optimal timing, filling and subsequent operation of reservoirs in hydro power systems.
3. Operational years and water years

There are two setups in HYENA, the static setup and the dynamic setup.

In the Static setup we have 1 operational year i.e. 2010 and say 51 water years say 1950-2000. We simulate over the 51 water year with the same demand (2010) every year. We start with specific reservoir content in the beginning of the first water year and the reservoir content in the end of one water year is the reservoir content in the beginning of next water year.

In the Dynamic setup we have >1 operational years i.e. 2004-2014. The Power System is simulated by the following schema:

|----|------|------|------|------|------|------|------|------|------|------|------|

We start with specific reservoir content in the beginning of the first operational year 2004 and start with water year 1950. The reservoir content in the end of the year is the reservoir content in the beginning of operational year 2005 and water year 1951. We move like that horizontally until we come to the end of the row. Next we begin in second row with operational year 2004 and start with water year 1951 and continue horizontally throughout the rest of the schema. Water years are cycled so after the last water year 2000 comes again the first water year 1950.

In the Static setup is equivalent to moving down one column starting with the first water year 1950.

4. Strategic part

Water values are calculated in the strategic part.

First the Power System is subdivided into water areas. In each water area all hydro and geothermal stations are modelled together as a single power station with a single upstream reservoir. Substation with reserve stations and electricity market are placed within water areas. See figure 4.
Water values are expressed as a function of reservoir content in the water area and time.

\[ \alpha(v_{wa}, t) \]

Calculation of water values is explained in figure 5. The vertical axis is divided into intervals and \( V_t \) is the reservoir content in one point. The single reservoir system is simulated one interval forward to time \( t+1 \) by the following model:

\[ Q_{t+1} = Q_t + \alpha \cdot R_t + (w - \min(w, U_t)) \]

\( Q_t \) is reservoir content in GWh, \( R_t \) is regulated inflow in GWh/week and \( U_t \) is unregulated inflow in GWh/week. \( \alpha \) is aversion factor when drawdown of reservoir results in lower head at power stations. This constant is an input parameter and can be determined by experiment. It is usually on the interval 0.6 – 1.0. No variable head means \( \alpha = 1,0 \).

In calculating water values at \( V_t \) we simulate the system for all instances of river flow say 51 and we can have three possible outcomes according to fig 4:

1. Reservoir full \( Q_{1,t+1} > Q_{\text{max}} \) then water value is 0.

2. Reservoir empty \( Q_{1,t+1} < 0 \) then water value is equal to the variable cost of the last reserve unit (or curtailment price or spot market price etc) used to lift the reservoir level over 0.

3. Reservoir between empty and full then water value is the same as the water value at \( V_{3,t+1} \) calculated by interpolation.

New water value at \( V_t \) is calculated by taking the average over all water years.

The whole procedure goes backward in time week by week or day by day and upwards from \( Q = 0 \) to \( Q = Q_{\text{max}} \) in increments of say 2%. Usually 3 rounds in the water value schema results in a satisfactory convergence.
5. Tactical part

The Multiobjective function for every stage $t$ of the planning period $T$ is:

$$
\min \left\{ \sum_{j \in H_{Res}} \varphi \cdot \frac{\partial P_j}{\partial v_j} \cdot \alpha_{s,t+1} \cdot u_{j,t}^2 \\
+ \sum_{j \in H_{ROR}} \xi \cdot \rho_k \cdot s_{j,t} \\
+ \sum_{j \in H_{All}} c_j \cdot g_{j,t} \\
- \sum_{j \in Sel} c_j \cdot S_{j,t} \\
+ \sum_{j \in Buy} c_j \cdot B_{j,t} \\
+ \sum_{j \in Short} c_j \cdot g_{j,t}^2 \\
+ \sum_{j \in Trans} \lambda \cdot L_{j,t}^+ + \bar{\lambda} \cdot L_{j,t}^- \\
- \beta \cdot \sum_k \int_0^{v_{k,t+1}} \rho_k(x) \cdot \alpha_{s,t+1}(x) \cdot dx \\
+ \sum_k \psi_a \cdot S_a \right\}
$$

Turbined water (Plants with Reservoirs)

Turbined water (Run-of-the-River)

Spilled water

Thermal

Selling on the Spot Market

Buying on the Spot Market

Power Shortage

Transmission

Reservoirs

Spinning Reserve requirements

Continuity equation for electricity in substations:

$$\sum_{i \in S} \rho_i(v_i) \cdot u_{i,t} + \sum_{j \in T,S} g_{j,t} - \sum_{j \in S} B_{j,t} + \sum_{j \in S} S_{j,t} - \sum_{n \in S} L_n^+ + \sum_{n \in S} L_n^- - a \cdot L_n^{-2} - b \cdot L_n^- - c = w_{s,t}$$

Hydro Production

Thermal Production and Shortage

Buying on the Spot Market

Selling on the Spot Market

Transmission from substation

Transmission to substation

Load in substation
Water balance for every hydro power station; continuity equation for water:

\[ v_{i,t+1} = v_{i,t} - u_{i,t} - s_{i,t} + r_{i,t} + \sum_{m \in U_i} [u_{m,t} + s_{m,t}] \]  

(3)

Limits on reservoir storage:

\[ v_{i}^{\text{min}} \leq v_{i,t} \leq v_{i}^{\text{max}} \]  

(4)

Limits on turbined water:

\[ u_{i}^{\text{min}} \leq u_{i,t} \leq u_{i}^{\text{max}} \]  

(5)

Limits on spilled water:

\[ s_{i}^{\text{min}} \leq s_{i,t} \leq s_{i}^{\text{max}} \]  

(6)

Limits on thermal generation:

\[ 0 \leq g_{j,t} \leq g_{j}^{\text{max}} \]  

(7)

Curtailment of secondary energy:

- Max 50% every year
- Max 40% every 4 consecutive years
- Max 20% every 20 consecutive years

Fairness between I customers in curtailment of secondary energy.

\[ \sum_{j \in \text{Curt}} \frac{g_{j,t}}{g_{j}^{\text{max}}} = \text{Either } I \text{ or 0 } \text{(integer variable)} \]

Transmission line capacity:

\[ L_{n}^{+} \leq L_{n}^{\text{max}} \quad L_{n}^{-} \leq L_{n}^{\text{max}} \]  

(8)

Limits on transmission capacity in Flow Gates (FG):

\[ \sum_{n \in \text{FG}} f_{n}^{+} \cdot L_{n}^{+} + \sum_{n \in \text{FG}} f_{n}^{-} \cdot L_{n}^{-} \leq L_{a}^{+} \]

\[ \sum_{n \in \text{FG}} (1 - f_{n}^{+}) \cdot L_{n}^{+} + \sum_{n \in \text{FG}} (1 - f_{n}^{-}) \cdot L_{n}^{-} \leq L_{a}^{+} \]  

(9)

Spinning reserve requirement in hydro and geothermal plants for each area:

\[ \sum_{i \in a} \rho_{i}(v_{i}) \cdot u_{i,t} - S_{a} \leq (1 - \sigma_{a}) \cdot \sum_{i \in a} \rho_{i}(v_{i}) \cdot u_{i}^{\text{max}} \]  

(10)

Note: The definitions above refer to the HYENA Option: Mixed Integer Nonlinear Programming, with Transfer constraints in transmission lines and flowgates and Nonlinear losses. All other options in chapter 2 are subsets of this general option.
6. Definitions

t: time index
T: planning period
a: area index
s: subsystem index
c: reservoir index
f: thermal plant index
h: hydro plant index
m: index for upstream hydro plants
n: transmission line index
vi,t: Stored volume at plant i at the beginning of stage t
v_i,t+1: Stored volume at plant i at the end of stage t
ri,t: Lateral river flow arriving at power station i in stage t
ui,t: Turbined outflow at power station i in stage t
ui,t: Spilled outflow at power station i in stage t
Ui: Set of hydro plants immediately upstream of plant i
gj,t: Generation of thermal plant j in stage t
cj: Generation cost of thermal, shortage and prices on spot market
Bj: Buying on Spot Market (Defined for peak and off-peak load)
Sj: Selling on Spot Market
\sum_j c_j \cdot g_{j,t}: The immediate thermal operating cost in stage t
f: Factor in penalty function for turbined water (0.4-1.0)
ξ: Penalty for spilling water
λ: Damping factor for transmission (i.e. 0.00003)
\sum_k \rho_k(v_{r+1}) \cdot \alpha_{k+1} \cdot v_{k+1}: The future cost represented by:
\rho_k(v_{r+1}): The production coefficient of reservoir k [kWh/kl]
\alpha_{k,t+1}: The water value for subsystem k [kr/kWh]
v_{k,t+1}: Reservoir volume at the end of stage t [kl]
β: Discount factor
\sum_n^+ L_n: Transmission in positive direction between subsystems
\sum_n^- L_n: Transmission in negative direction between subsystems
τn: Transmission losses
L_{FG}: Limit on transmission in Flow Gates.
f_{n+}: =1 if direction of transmission line is the same as direction of Flow Gate, else=0.
f_{n-}: =1 if direction of transmission line is the opposite to direction of Flow Gate, else=0.
ψa: Penalty factor for spinning reserve requirement in area
Sa: Lack of spinning reserve
σa: Spinning reserve requirements, i.e. σ_week=0.10 and σ_day=0.075
w_{a,t}: Energy market in a subsystem
w_{a,t}: Energy market linked to a spinning reserve group.
7. Supply and Demand

In deregulated power systems market equilibrium is established where the supply (marginal cost) and demand curves meet, which defines competitive quantity on the market and market price as described in figure 6.

The demand curve represents that in nearly all markets quantity demanded rises as price falls; buyers wish to buy more at lower price and less at higher price. If the demand curve is vertical price changes will not affect the buying behavior of the consumer.

The supply curve represents that at lower prices, suppliers are less willing or capable of producing. The slope of the curve can be zero if suppliers are able because of their costs to supply more output at the same cost as with hydro power in Iceland in off critical periods.

Consumer surplus is the difference between the market price and what customers would have been willing to pay and producer surplus is the difference between the market price and at what price producers would have been willing to produce. Social surplus is the sum of consumer and producer surplus.

Figures 7-9 show examples of supply/demand curves and represents actual data calculated in the HYENA Simulation Model.

Figure 7 shows typical supply and demand curve for the South-West area of Iceland which has predominantly hydro power. Curtailment option is both included in the supply and demand curve.
Figure 8 shows supply and demand curves for a small power system connected to an active spot market. In this situation the customer is buying from the market and Hydro Power has been diminished to a minimum to collect water in reservoirs.

Figure 8

Figure 9 shows for the same system a situation with much more Hydro Power Production a part of which is sold on the market (Sell Peak).

Figure 9